Nir Regev, Principal Scientist

AlephZero Consulting, LLC

September 2018

On The Sensitivity of DOA Estimation to Baseline Inaccuracies

The following paper shows the sensitivity of DOA algorithms to the array baseline length. It is shown that a minor error (1.8mm @5GHz) can cause up to 3° of error in the DOA estimation. Thus, an algorithm for estimating the correct baseline is proposed and results are given.

Notations and assumptions

In Figure 1 a linear equally spaced array is depicted with baseline d and a single far field target with DOA denoted by θ. The radar is illuminating the search volume with a known transmitted signal.

Problem formulation

Since the target is in the far field it can be easily shown that the array manifold is given by:

$$\mathbf{a}\left(\theta\right) = \left(1, e^{-j\frac{2\pi}{\lambda}d\sin\theta}, \cdots, e^{-j\frac{2\pi}{\lambda}(N-1)d\sin\theta}\right)^T, \tag{1}$$

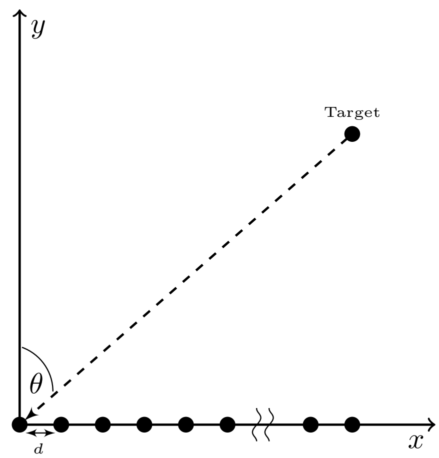

Figure 1: Linear array with baseline d and a target in the far field arriving from direction θ

where N is the number of elements in the array and λ is the wavelength.

Hence, the baseband received signal model in the time domain (or equivalently in the frequency domain) is given by:

$$\mathbf{x}_k = \mathbf{a}\left(\theta\right) s_k + \mathbf{w}_k, \tag{2}$$

where $k = 1, \cdots, K$ is the observation index and K is the number of observations, $\mathbf{w}_k$ is a circularly complex zero mean Gaussian noise with covariance matrix $\sigma_w^2 \mathbf{I}_N$, $\mathbf{I}_N$ is an $N \times N$ identity matrix.

Now, the problem of finding the direction of arrival θ is called the DOA problem and is of great importance in countless applications.

Let us formulate (2) with emphasis on the baseline dependence

$$\mathbf{x}_k = \mathbf{a}\left(\theta, d\right) s_k + \mathbf{w}_k, \tag{3}$$

Obviously, if the baseline, d is inaccurate the DOA estimation will suffer, more in large θ's than in DOAs that tend to zero as will be shown in the next section.

Illustration of baseline influence on DOA estimation

In Figures 1-3 we assumed the following:

1. $N = 30$

2. $d = \frac{\lambda}{3}$

3. $SNR = 50dB$ to yield errors that are noise independent.

4. In Figure 2 the receiver assumes a baseline value of the following form $d' = \alpha d$ where $\alpha = 0.9850, 0.9900, 0.9950, 1.0000, 1.0050, 1.0100, 1.0150$ and is changed from one run to the next. i.e. the wrong baseline d' deviates from -1.5% to $+1.5\%$ of the real baseline value d.

5. In Figure 3 the receiver assumes a baseline value of the following form $d' = \alpha d$ where $\alpha = 0.95, 1.00, 1.05$ and is changed from one run to the next. i.e. the wrong baseline d' deviates from -5% to $+5\%$ of the real baseline value d.

6. The DOA is estimated using MUSIC algorithm with polynomial rooting [1].

7. The DOA θ is changed from $-70°$ to $+70°$.

The Figures below illustrate the acute influence the accuracy of the baseline has on DOA estimation. Just for illustration if $\lambda = 6cm$ ($f_c = 5GHz$) a 3% deviation is just an error of $1.8mm$ in baseline position. Hence, a calibration algorithm to determine the correct baseline is proposed.

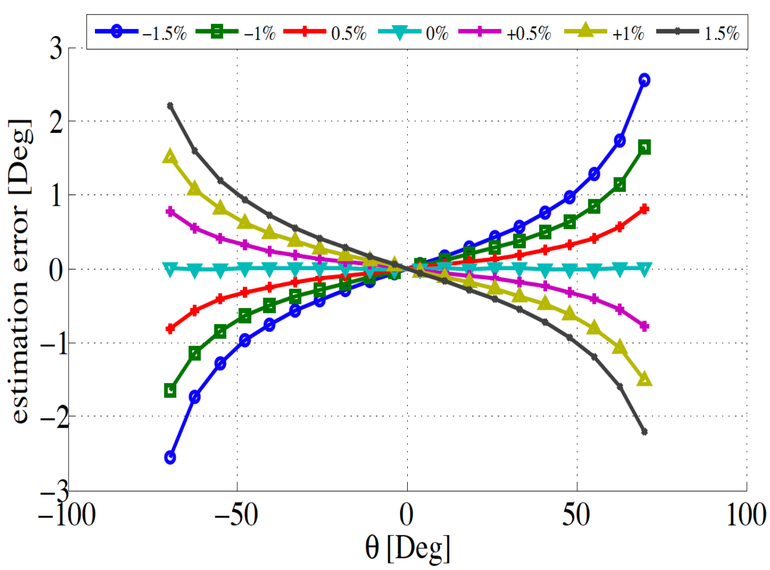

Figure 2: Error in DOA estimation

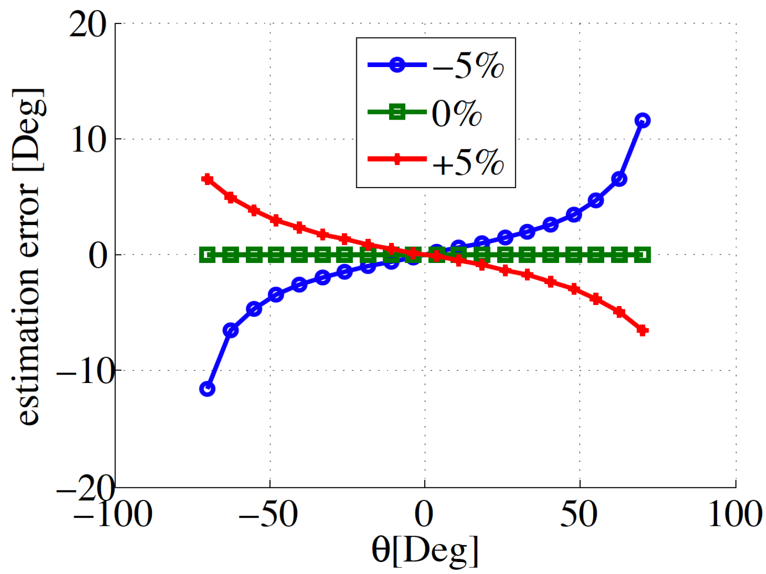

Figure 3: Error in DOA estimation

Estimating/Calibrating the Baseline

The calibration will be done in a controlled environment like an anechoic chamber, where K array samples will be generated with a known source (e.g. known BPSK sequence) for each L angles denoted by θ in the range $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Using (3) we can estimate the parameter of interest d. Thus, the Maximum Likelihood estimator is given by:

$$\hat{d}_{ML} = \arg\left\{\min_{d} \|\mathbf{x}_k - \mathbf{a}\left(\theta, d\right) s_k\|_2^2\right\}. \tag{4}$$

As a simple example, we take the case of two elements array in which the baseline d can be directly calculated from:

$$\hat{d}_{ML} = \frac{1}{L}\sum_{i=1}^{L} \hat{d}_i, \tag{5}$$

where $\hat{d}_i$ is given by

$$\hat{d}_i = \left|\frac{\varphi_i}{\lambda 2\pi \sin\theta}\right|, \tag{6}$$

where φ_i is obtained by calculating the mean of the electronic angle difference measured on the elements over K samples. This procedure is repeated L times for different DOAs $\theta_i,\ i = 1, \cdots, L$ and averaged to obtain the final result.

References

[1] H. K. Hwang, Z. Aliyazicioglu, M. Grice, and A. Yakovlev, "Direction of arrival estimation using a root-music algorithm," in *Proceedings of the International MultiConference of Engineers and Computer Scientists 2008 Vol II IMECS 2008, 19-21 March, 2008, Hong Kong.*